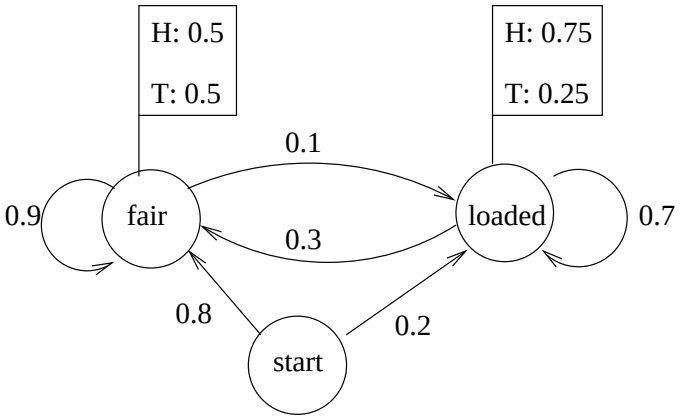

The probability that the model picks the path FFF and generates HHT is

$(0.8)(0.5)(0.9)(0.5)(0.9)(0.5) = 0.081$,

while the probability that it picks the path LLL and generates HHT is

$(0.2)(0.75)(0.7)(0.75)(0.7)(0.25) = 0.0137\cdots$.

To get the probability that HHT is generated by the model, we need to sum, over all 8 paths of length 3, the probability of picking that path and generating HHT.

Example of Viterbi Decoding. For observed output, such as:

TTHTHTTHTHTTHHTHTHHTHHHHHTHHTHHTTHTTHTHTTHTTTHTT

what is the most likely state path (sequence of states) to have generated the observed sequence? More precisely, we want the path that maximizes the joint probability of picking the path and generating the observed sequence?

Let π denote an arbitrary path of length 3 in the "coin" HMM. Fix S as the observed sequence HHT. Let $P(\pi, S)$ denote the joint probability of π and S, i.e., the probability of picking π and generating S.

π	$P(\pi, S)$
FFF	0.081
FFL	0.0045
FLF	0.0045
FLL	0.00525
LFF	0.010125
LFL	$0.000562\cdots$
LLF	$0.011812\cdots$
LLL	$0.013781\cdots$
total	$0.131449\cdots$

Thus, the probability of S (given the model) is $P(S) = 0.131449\cdots$.

Given that the observed sequence HHT was generated, what is the "posterior" probability, $P(\pi|S)$, that a given state-path π was taken? We use the formula

$$P(\pi|S) = P(\pi, S)/P(S)$$

π	$P(\pi, S)$	$P(\pi\|S)$
FFF	0.081	$0.6158\cdots$
FFL	0.0045	$0.0342\cdots$
FLF	0.0045	$0.0342\cdots$
FLL	0.00525	$0.0399\cdots$
LFF	0.010125	$0.0769\cdots$
LFL	$0.000562\cdots$	$0.0427\cdots$
LLF	$0.011812\cdots$	$0.0898\cdots$
LLL	$0.013781\cdots$	$0.1047\cdots$

Thus the most probable path is FFF.

What is the probability that the second H in HHT was generated in the F state?

$0.6158 + 0.0342 + 0.0769 + 0.0427 \approx 0.77$